Journal of Applied Mechanics and Technical Physics, Vol. 42, No. 5, pp. 864–871, 2001

CONSTITUTIVE EQUATIONS FOR TWO-STEP THERMOELASTIC PHASE TRANSFORMATIONS

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UDC 539.4

A number of hypotheses on the mechanical behavior of shape memory alloys such as titanium nickelide in two-step (martensitic and rhombohedral) phase transformations are formulated on the basis of experimental data. A system of relations linking stresses, strains, temperature, and phase composition in such transitions is proposed.

As is known, besides a thermoelastic phase transition to the martensitic state with a monoclinic crystal lattice denoted by B19′ (below M-transition), titanium nickelide can undergo a transition from the austenitic phase with a body centered cubic lattice B2 to the rhombohedral phase R and back (below R-transformation).

Experimental data on the mechanical behavior of materials during the R-transition and the two-step R- and M-transformation are given in [1–11] and other papers. These data lead to conflicting conclusions. Thus, Stachoviac and McCormic [7], using experimental data, make the paradoxical conclusion that the maximum strain generated in the direct R-transformation decreases with increase in applied stress. At the same time, according to the data of [9, 10], the strain due to the complete direct R-transformation increases with increase in applied stress. According to [9], the reverse transformation is a one-step process, and from the data of [10], it is a two-step process. According to the data of [7], the reverse transformation from the two-phase state proceeds in two steps; otherwise, it proceeds in one step.

In the present work, from analysis of experimental data, we formulated a number of simplifying assumptions on two-step phase transformations in titanium nickelide. A system of constitutive relations linking phase composition, temperature, stress, and strain in such processes is proposed.

1. Qualitative Description of the Two-Step Phase Transformation. Having analyzed experimental data, we propose a model for the two-step phase transition in titanium nickelide, which is based on the following hypotheses.

The direct and reverse phase transformations imply transformations in which the degree of crystal-lattice symmetry decreases or increases, respectively. Thus, the transitions B2 → R, B2 → B19′, and R → B19′ are direct, and the transitions B19′ → R, B19′ → B2, and R → B2 are reverse. Transitions from the same phase to different final states can be compared in the degree of lattice symmetry. Thus, the transition B2 → B19′ changes symmetry more radically than B2 → R, and B19′ → B2 changes it more radically than R → B2.

1. For the direct transformations, the following assumptions are adopted:

1.1. The temperature–force conditions of the beginning and end of formation of a new phase do not depend on the type of structure from which this phase is formed.

1.2. If the same initial phase can simultaneously undergo two direct phase transitions which differ in the final product, the transition occurs that changes the crystal lattice symmetry more radically.

1.3. If there are simultaneous phase transitions from two different structures to the same structure, these transitions do not change the relation between parameters of these structures.

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0021-8944/01/4205-0864 $25.00

2. For the reverse transformation, the following assumptions are adopted:

2.1. The dependences of the temperature of the beginning and end of a transition from a certain initial phase on stress are identical no matter what the product of this transition.

2.2. If temperature–force conditions allow reverse transformations from the same phase to two different phases between which a phase transition is impossible under the given conditions, the transformation occurs that changes crystal symmetry to a lesser extent.

2.3. If reverse phase transitions from a low-symmetry phase to an intermediate phase and from the intermediate phase to a more symmetric phase are possible simultaneously, the intermediate transformation in the constitutive equations can be ignored, assuming that there is immediate transition of both the first and the second phases to the final phase.

Hypotheses 1.1, 1.2, 2.1, and 2.2 are based on analysis of the experimental data of [1–11]. Assumptions 1.3 and 2.3 are adopted to simplify the constitutive relations.

According to assumptions 1.1 and 2.1, there are eight characteristic temperatures of the phase transformations: $\mathrm{R}^{+}_{\text{begin}}$ and $\mathrm{R}^{+}_{\text{end}}$ are the temperatures of the beginning and end of formation of the rhombohedral phase in the direct transformation, $\mathrm{M}^{+}_{\text{begin}}$ and $\mathrm{M}^{+}_{\text{end}}$ are the temperatures of the beginning and end of formation of the martensitic phase in the direct transformation [without regard to the particular phase (austenitic or rhombohedral) from which the martensitic phase is formed]; $\mathrm{M}^{-}_{\text{begin}}$ and $\mathrm{M}^{-}_{\text{end}}$ are the temperature of the beginning and end of transition from the martensitic phase in the reverse transformation [without regard to the particular phase (rhombohedral or austenitic) produced by the transition]; $\mathrm{R}^{-}_{\text{begin}}$ and $\mathrm{R}^{-}_{\text{end}}$ are the temperatures of the beginning and end of transition from the rhombohedral phase in the reverse transformation.

In describing two-step thermoelastic transformations for both direct and reverse transitions, it is common to introduce the temperatures of the beginning and end of formation of the final reaction products. Thus, constructing a consistent model within the framework of linear dependences of transition temperatures on stress is not possible. Indeed, for small stresses, the austenitic phase is formed from the rhombohedral phase in the reverse transformation. Therefore, in this region, the lines of the beginning and end of formation of the austenitic phase should correspond to the lines of the beginning and end of formation of the rhombohedral phase in the direct transformation. At the same time, for large stresses, the austenitic phase is formed from the martensitic phase in the reverse transformation. These lines should correspond to the straight lines of the beginning and end of formation of the martensitic phase in the direct transformation. Therefore, the lines of the beginning and end of formation of the austenitic phase in the reverse transformation should have inflection, and at the point of inflection, the slope of the lines increases by about a factor of three.

A fundamental difference between the proposed system of characteristic temperatures and conventional temperatures is that for the reverse transformation, we use the temperatures of disappearance of the initial structure instead of the temperatures of formation of the end product. The lines of disappearance of the rhombohedral (martensitic) phase in the reverse transformation correspond to the lines of formation of the rhombohedral (martensitic) phase in the direct transformation. Therefore, in transition from large to small stresses, these lines have no inflections.

2. Constitutive Equations of Phase-Composition Variation. Below, according to [9], it is assumed that the transition temperatures are linear functions of the stress intensity σ_i:

$$\mathrm{R}^{+}_{\text{begin}} = \mathrm{R}^{0+}_{\text{begin}} + k^{+}_{\mathrm{R}}\sigma_i, \qquad \mathrm{R}^{+}_{\text{end}} = \mathrm{R}^{0+}_{\text{end}} + k^{+}_{\mathrm{R}}\sigma_i, \qquad \mathrm{R}^{-}_{\text{begin}} = \mathrm{R}^{0-}_{\text{begin}} + k^{-}_{\mathrm{R}}\sigma_i, \qquad \mathrm{R}^{-}_{\text{end}} = \mathrm{R}^{0-}_{\text{end}} + k^{-}_{\mathrm{R}}\sigma_i,$$
$$\mathrm{M}^{+}_{\text{begin}} = \mathrm{M}^{0+}_{\text{begin}} + k^{+}_{\mathrm{M}}\sigma_i, \quad \mathrm{M}^{+}_{\text{end}} = \mathrm{M}^{0+}_{\text{end}} + k^{+}_{\mathrm{M}}\sigma_i, \quad \mathrm{M}^{-}_{\text{begin}} = \mathrm{M}^{0-}_{\text{begin}} + k^{-}_{\mathrm{M}}\sigma_i, \quad \mathrm{M}^{-}_{\text{end}} = \mathrm{M}^{0-}_{\text{end}} + k^{-}_{\mathrm{M}}\sigma_i, \qquad (2.1)$$
$$k^{-}_{\mathrm{R}} \approx k^{+}_{\mathrm{R}} = k_{\mathrm{R}}, \qquad k^{-}_{\mathrm{M}} \approx k^{+}_{\mathrm{M}} = k_{\mathrm{M}}, \qquad k_{\mathrm{M}} \approx 3k_{\mathrm{R}}.$$

For simplicity, the effect of the first and third invariants of the stress tensor on the transition temperatures are ignored in the present work.

For titanium nickelide in which the concentration of nickel is 0.2% higher than its concentration in the equiatomic composition, according to the experimental data of [9]

$$\mathrm{R}^{0+}_{\text{begin}} = 46^\circ\mathrm{C}, \qquad \mathrm{R}^{0+}_{\text{end}} = 38^\circ\mathrm{C}, \qquad \mathrm{R}^{0-}_{\text{begin}} = 42^\circ\mathrm{C}, \qquad \mathrm{R}^{0-}_{\text{end}} = 50^\circ\mathrm{C},$$
$$\mathrm{M}^{0+}_{\text{begin}} = 5^\circ\mathrm{C}, \qquad \mathrm{M}^{0+}_{\text{end}} = -30^\circ\mathrm{C}, \qquad \mathrm{M}^{0-}_{\text{begin}} = 35^\circ\mathrm{C}, \qquad \mathrm{M}^{0-}_{\text{end}} = 45^\circ\mathrm{C}, \qquad (2.2)$$
$$k_{\mathrm{R}} = 0.073, \qquad k_{\mathrm{M}} = 0.264.$$

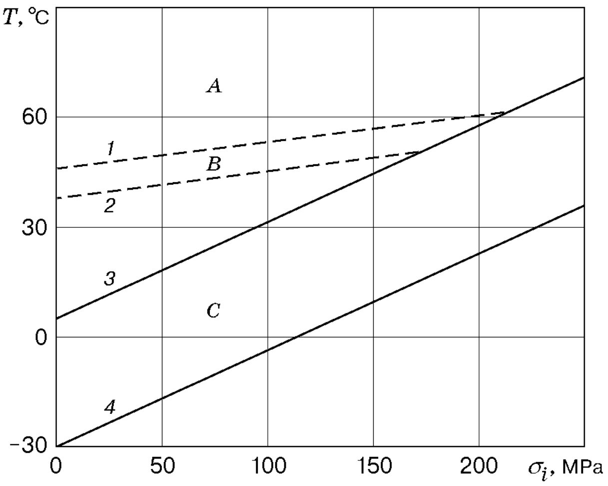

Fig. 1

We introduce the three internal state variables (q_{M}, q_{A}, and q_{R}), which can be treated as the volume fraction of the martensitic and austenitic phases, and the degree of completeness of the R-transformation. The last quantity is considered normalized:

$$q_{\mathrm{A}} + q_{\mathrm{R}} + q_{\mathrm{M}} = 1. \tag{2.3}$$

The lines of the beginning and end of the direct transformations corresponding to dependences (2.1) and data (2.2) are shown in Fig. 1 in the σ_i–T coordinates (the dashed straight lines correspond to the rhombohedral transformation, and the solid straight lines correspond to the martensitic transformation). In the region A (Fig. 1) no direct phase transitions occur. Let the region B be part of the rhombohedral strip between the straight lines 1 $[T = \mathrm{R}^{+}_{\mathrm{begin}}(\sigma)]$ and 2 $[T = \mathrm{R}^{+}_{\mathrm{end}}(\sigma)]$ that does not belong to the martensitic region C lying between the straight lines 3 $[T = \mathrm{M}^{+}_{\mathrm{begin}}(\sigma)]$ and 4 $[T = \mathrm{M}^{+}_{\mathrm{end}}(\sigma)]$. The conditions of the direct rhombohedral transformation are the following: 1) presence of the austenitic phase in the material; 2) the image point should be present in the region B; 3) the motion of the image point should satisfy the inequality

$$k_{\mathrm{R}}\, d\sigma_i > dT. \tag{2.4}$$

The degree of completeness of the rhombohedral transformation varies in accordance with the dependence

$$q_{\mathrm{R}} = 1 - (1 - q^0_{\mathrm{R}})(1 - f(\mathrm{R}^{+}_{\mathrm{begin}}, \mathrm{R}^{+}_{\mathrm{end}}, T)), \tag{2.5}$$

where q^0_{R} is the value of q_{R} when the image point intersects the left (upper) boundary of the rhombohedral region. If we consider a step of the rhombohedral transformation that begins from a point inside the rhombohedral strip, then, instead of $\mathrm{R}^{+}_{\mathrm{begin}}$ as the first argument of the function f (2.5), we should use the temperature T^{+}_{begin} beginning with which condition (2.4) is satisfied in the process considered, and the value of q^0_{R} at this temperature should be used as q_{R}. Formula (2.5) is based on the assumption of similarity of direct-transformation curves with temperature variation in incomplete intervals of phase-transition temperatures with respect to the end point of the direct transformation [12–14].

According to [14], the function f can be defined as

$$f(T_1, T_2, T) = \sin\Big(\frac{\pi}{2}\,\frac{T_1 - T}{T_1 - T_2}\Big). \tag{2.6}$$

From experimental results of [2, 3, 7, 9, 10] it follows that phase-composition variations correlate with variations in relative electrical resistance, whose temperature dependence can be described by formulas of the type of (2.6).

Since in the direct transformation, only the austenitic phase can transform into the rhombohedral phase, the fraction of the martensitic phase for the process considered has constant value $q_{\mathrm{M}} = q^0_{\mathrm{M}}$, and, by virtue of (2.3), the fraction of the austenitic phase is determined from the relation

$$q_{\mathrm{A}} = 1 - q_{\mathrm{R}} - q^0_{\mathrm{M}}. \tag{2.7}$$

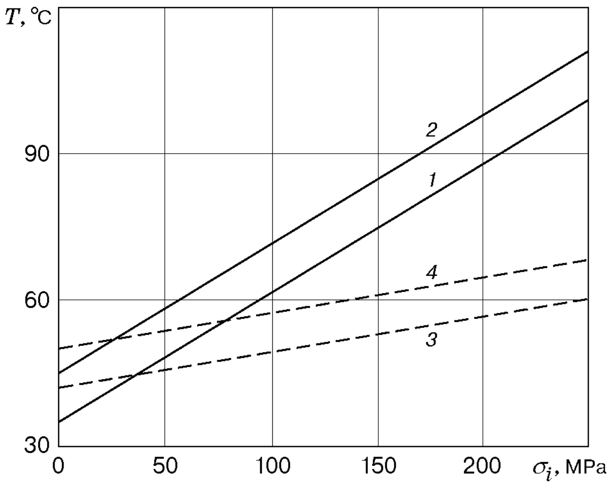

Fig. 2

If $q_{\rm M}^0 \neq 0$, then the change of $q_{\rm R}$ according to (2.5) does not stop with intersection of the right (lower) boundary of the rhombohedral region but it continues until the quantity $q_{\rm A}$ calculated from (2.7) becomes equal to zero. After this, the phase transitions stop until the beginning of the martensitic transformation. The direct rhombohedral transformation also stops, without having completed, if the image point falls in the common part of the martensitic and rhombohedral regions, which corresponds to assumption 1.2.

Sufficient conditions of the direct martensitic transformation are the presence of the austenitic or rhombohedral phases in the material, the presence of the image point in the martensitic region, and satisfaction of the following inequality for increments during motion of the image point:

$$k_{\rm M}\, d\sigma_i > dT. \tag{2.8}$$

The increase in the fraction of the martensitic phase obeys the dependence

$$q_{\rm M} = 1 - (1 - q_{\rm M}^0)(1 - f({\rm M}_{\rm begin}^+, {\rm M}_{\rm end}^+, T)), \tag{2.9}$$

where $q_{\rm M}^0$ is the value of $q_{\rm M}$ when the image point intersects the left (upper) boundary of the martensitic region. If a step of the direct transformation begins at a point located inside the martensitic region, the instead of ${\rm M}_{\rm begin}^+$ as the first argument of the function f (2.9), we should use the temperature $T_{\rm begin}^+$, beginning with which condition (2.8) is satisfied for the process considered.

During the direct transformation, both the austenitic and rhombohedral phases can transform into the martensitic phase. Let, at the moment of beginning of the direct martensitic transformation, $q_{\rm R} = q_{\rm R}^0$ and $q_{\rm A} = q_{\rm A}^0$. By virtue of assumption 1.3, in the transition considered, the ratio between the parameters of the initial phases is conserved:

$$q_{\rm R}/q_{\rm A} = q_{\rm R}^0/q_{\rm A}^0. \tag{2.10}$$

In addition,

$$q_{\rm R} + q_{\rm A} = 1 - q_{\rm M}. \tag{2.11}$$

Solving system (2.10), (2.11), taking into account that $q_{\rm R}^0 + q_{\rm A}^0 = 1 - q_{\rm M}^0$ and using formula (2.9), we obtain

$$q_{\rm R} = q_{\rm R}^0(1 - f_1({\rm M}_{\rm begin}^+, {\rm M}_{\rm end}^+, T)), \qquad q_{\rm A} = q_{\rm A}^0(1 - f_1({\rm M}_{\rm begin}^+, {\rm M}_{\rm end}^+, T)). \tag{2.12}$$

Relations (2.5), (2.9), and (2.12) define the variation of the phase composition in the direct two-step transformation.

Figure 2 shows the lines describing the reverse transformation corresponding to dependences (2.1) and data (2.2) (continuous straight lines correspond to the martensitic transformation and the dashed straight lines correspond to the rhombohedral transformation). The conditions of the reverse martensitic transformation are the presence of the image point in the strip between the straight lines 1 [$T = {\rm M}_{\rm begin}^-(\sigma)$] and 2 [$T = {\rm M}_{\rm end}^-(\sigma)$] (Fig. 2) and satisfaction of the inverse of inequality (2.8). The condition of the reverse rhombohedral transformation is the presence of the image point in the region between the dashed straight lines 3 [$T = \mathrm{R}^{-}_{\mathrm{begin}}(\sigma_i)$] and 4 [$T = \mathrm{R}^{-}_{\mathrm{end}}(\sigma_i)$] (Fig. 2) and satisfaction of the inequality inverse of (2.4).

In the reverse transformation, the decrease in the parameter of the initial phase is given by the dependences

$$q_{\mathrm{M}} = q^0_{\mathrm{M}} f(\mathrm{M}^{-}_{\mathrm{end}}, \mathrm{M}^{-}_{\mathrm{begin}}, T); \tag{2.13}$$

$$q_{\mathrm{R}} = q^0_{\mathrm{R}} f(\mathrm{R}^{-}_{\mathrm{end}}, \mathrm{R}^{-}_{\mathrm{begin}}, T). \tag{2.14}$$

If the parameter of the martensitic or rhombohedral phases begins to decrease inside the corresponding region, as the first argument of the function f (2.13) or (2.14), we should use the temperature T^{-}_{begin} at which the inequalities for increments are satisfied for the first time, which corresponds to the beginning of the reverse transformation. In (2.13) and (2.14), q^0_{M} and q^0_{R} are the values of the corresponding parameters at the beginning of this step of the reverse transformation. Formulas (2.13) and (2.14) are obtained from the assumption that the reverse-transformation curves are similar with temperature variation in incomplete intervals of transition temperatures with respect to the end point of the reverse transformation [13, 14].

It should be noted that in the case of heating and simultaneous increase in stress intensity $k_{\mathrm{M}} > k_{\mathrm{R}}$, a situation is possible where the direct-transition conditions (2.8) are satisfied for the martensitic transformation, and the reverse-transition conditions are satisfied for the rhombohedral transformation. In addition, a situation is possible where under cooling and decrease in stress intensity, the reverse martensitic transformation and the direct rhombohedral transformation can occur simultaneously. However, in the present work, such cases are not considered.

Formulas (2.13) and (2.14) are valid irrespective of the end product of the phase transition. If the R-strip is intersected during the reverse transformation and there is no simultaneous intersection of the M-strip, the rhombohedral phase can transform only to the austenitic phase. Therefore, the quantity q_{M} has a constant value: $q_{\mathrm{M}} = q^0_{\mathrm{M}}$. The quantity q_{R} decreases in accordance to the dependence (2.14), and, thus, $q_{\mathrm{A}} = 1 - q_{\mathrm{R}} - q^0_{\mathrm{M}}$. If there is intersection of the martensitic-transformation line and there is no intersection of the rhombohedral region, the end product depends on where the image point is located — above or below the rhombohedral region. If it is lower (small stresses), the parameter of the austenitic phase has a constant value $q_{\mathrm{A}} = q^0_{\mathrm{A}}$, the parameter of the martensitic phase decreases in accord with (2.13), and $q_{\mathrm{R}} = 1 - q_{\mathrm{M}} - q^0_{\mathrm{A}}$. The process is described by these relations until the image point leaves the strip of the martensitic transformation or until the value of q_{R} becomes zero. If the point of entry into the martensitic strip is above the rhombohedral region, the martensitic phase decreases in accord with (2.13), $q_{\mathrm{R}} = 0$, and $q_{\mathrm{A}} = 1 - q_{\mathrm{M}}$.

If during the reverse transformation, the image point of the state of the material, moving, intersects simultaneously the rhombohedral and martensitic regions, the change of the parameters q_{M} and q_{R} is given by formulas (2.13) and (2.14), and the quantity q_{A} is defined by the relation $q_{\mathrm{A}} = 1 - q_{\mathrm{R}} - q_{\mathrm{M}}$.

3. Constitutive Equations for Phase Strains. The system describing the development of phase strains is written as follows:

— for the direct transformation,

$$\varepsilon_{ij} = \varepsilon^{\mathrm{R}}_{ij} + \varepsilon^{\mathrm{M}}_{ij}, \qquad d\varepsilon^{\mathrm{R}}_{ij} = c^{\mathrm{R}}_0 \sigma'_{ij}\, dq_{\mathrm{R}}, \qquad d\varepsilon^{\mathrm{M}}_{ij} = (\beta\delta_{ij} + c^{\mathrm{M}}_0 \sigma'_{ij} + a^{\mathrm{M}}_0 \varepsilon^{\mathrm{M}}_{ij}) dq_{\mathrm{M}};$$

— for the reverse transformation,

$$d\varepsilon^{\mathrm{R}}_{ij} = \Big(\frac{\varepsilon^{\mathrm{R}}_{0ij}}{q^{\mathrm{R}}_0}\Big) dq_{\mathrm{R}}, \qquad d\varepsilon^{\mathrm{M}}_{ij} = \Big(\frac{a^{\mathrm{M}}_0 \varepsilon^{\mathrm{M}}_{0ij}}{\exp(a^{\mathrm{M}}_0 q^{\mathrm{M}}_0) - 1} + a^{\mathrm{M}}_0 \varepsilon^{\mathrm{M}}_{ij}\Big) dq_{\mathrm{M}}.$$

Here for simplicity, we ignore the reversible shape memory effect in the reverse transformations and the volume effect of the reaction for the direct rhombohedral transformation; ε_{ij}, $\varepsilon^{\mathrm{M}}_{ij}$, and $\varepsilon^{\mathrm{R}}_{ij}$ are the total phase strain and the phase strain due to the martensitic and rhombohedral transformations (all phase strains are reckoned from the austenitic state), q^{R}_0, and $\varepsilon^{\mathrm{R}}_{0ij}$ are the values of the parameter of the rhombohedral phase and the corresponding phase strain at the initial point of the examined step of the reverse transformation, q^{M}_0 and $\varepsilon^{\mathrm{M}}_{0ij}$ are similar values for the reverse martensitic transition, and β, a^{M}_0, c^{R}_0, and c^{M}_0 are parameters of the material. The strain equations for the martensitic transformation are obtained in [12–14] using a micromechanical scheme that simulates simultaneously the processes of origin and growth of martensite crystals in the austenitic matrix. The strain equations for the R-transformation are written by analogy taking into account that the strain of the complete direct R-transformation under the action of constant stress is proportional to the magnitude of this stress, and the oriented transformation [15] is less pronounced for the rhombohedral transition than for the martensitic transformation [9]. For the martensitic

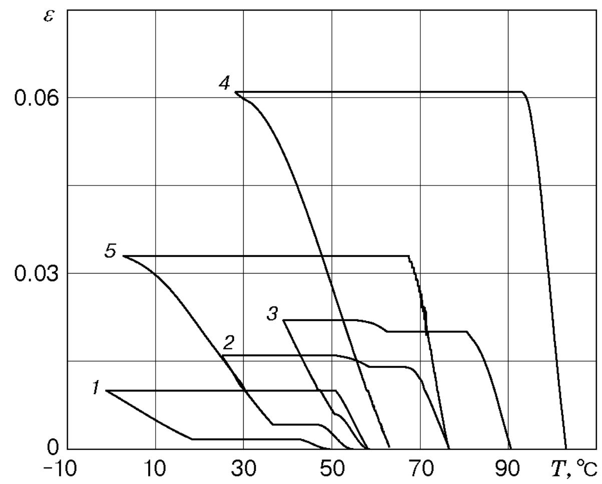

Fig. 3

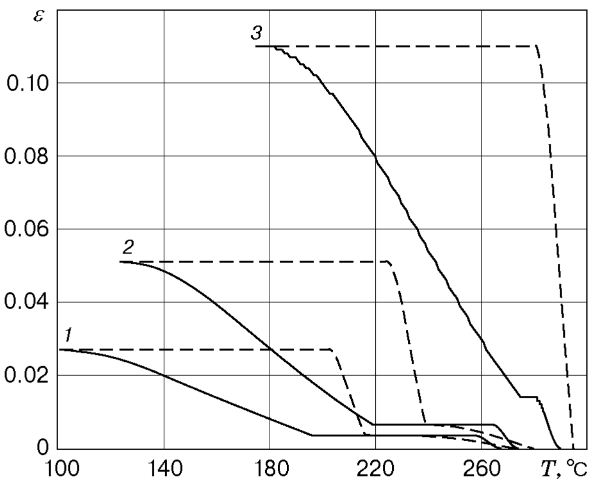

Fig. 4

transformation [15] of TN-1 titanium nickelide, $\beta = 0.00117$, $a_0^{\rm M} = 0.718$, $c_0^{\rm M} = 0.000283$ a MPa^{-1}, and for the rhombohedral transformation, $c_0^{\rm R} = 4.5 \cdot 10^{-5}$ MPa^{-1} [9].

4. Use of the Model to Describe the Behavior of Titanium Nickelide. Figure 3 shows the temperature dependences of the axial phase strain ε for the direct and reverse transformations under the action of constant uniaxial stress σ calculated from the equations described above for the transition temperatures (2.2) typical of Ti–Ni alloy (50.2% Ni). Let σ_k ($k = 1, 2, \ldots, 6$) be the abscissa of the points of intersection of the straight lines in Figs. 1 and 2:

$$\sigma_1 = \frac{{\rm R}_{\rm end}^{0+} - {\rm M}_{\rm begin}^{0+}}{\Delta k}, \qquad \sigma_2 = \frac{{\rm R}_{\rm begin}^{0+} - {\rm M}_{\rm begin}^{0+}}{\Delta k}, \qquad \sigma_3 = \frac{{\rm R}_{\rm end}^{0-} - {\rm M}_{\rm end}^{0-}}{\Delta k}, \qquad \sigma_4 = \frac{{\rm R}_{\rm end}^{0-} - {\rm M}_{\rm begin}^{0-}}{\Delta k},$$

$$\sigma_5 = \frac{{\rm R}_{\rm begin}^{0-} - {\rm M}_{\rm begin}^{0-}}{\Delta k}, \qquad \sigma_6 = \frac{{\rm R}_{\rm begin}^{0-} - {\rm M}_{\rm end}^{0-}}{\Delta k}, \qquad \Delta k = k_{\rm M} - k_{\rm R}.$$

For materials with typical transition temperatures (2.2), the inequalities $\sigma_6 < \sigma_3 < \sigma_5 < \sigma_4 < \sigma_1 < \sigma_2$, and $\sigma_6 < 0$ are satisfied.

For all the loops shown in Fig. 3, except for the loop 5, the temperature of the end of the cooling process is such that at the beginning of the reverse transformation, the material is in the two-phase state at $q_{\rm R} = q_{\rm M}$. For the loop 1, $\sigma = 50$ MPa $< \sigma_4$. In this case (at small stresses), only the direct transformation is a two-step process, and a pronounced anhysteretic segment is observed. For loop 2, $\sigma = 120$ MPa ($\sigma_4 < \sigma < \sigma_1$). In this case, both the direct and reverse transformations are two-step processes, and in both cases, the small-strain segment precedes the large-strain segment. For loop 3, $\sigma = \sigma_1 = 172$ MPa, and the anhysteretic segment on the direct transformation curve degenerates into a point of inflection. For loop 4, $\sigma = \sigma_2 = 214$ MPa, and both the direct and reverse transformations are one-step processes because the direct transformation does not produce the R-phase. Loop 5 corresponds to the case $\sigma = 120$ MPa, i.e., to the same stress as for loop 2 but the direct transformation has led to a completely martensitic state, and, hence, the reverse transformation occurred from a single-phase state. In this case, for any values of the applied stress, the reverse transformation has the one-step character.

However, for alloys with higher nickel concentration or with iron additives, two-step reverse transformations can also occurred from completely martensitic states because with increase in nickel concentration or with addition of iron, the martensitic-transition temperature decreases sharply. As a result, it may happen that ${\rm M}_{\rm end}^{0-} < {\rm R}_{\rm begin}^{0-}$ and $\sigma_6 > 0$. For small stresses in the reverse transformation from the completely martensitic state, the large-strain transformation B19$'$ → R first occurs, which is followed by an anhysteretic segment and then by the small-strain transition R → B2.

Figure 4 shows hysteresis loops obtained under heating (solid curves) and cooling (dashed curved) of the sample from a material with characteristics ${\rm R}_{\rm begin}^{0+} = 260$ K, ${\rm R}_{\rm end}^{0+} = 250$ K, ${\rm M}_{\rm begin}^{0+} = 170$ K, ${\rm M}_{\rm end}^{0+} = 70$ K, ${\rm M}_{\rm begin}^{0-} = 175$ K, ${\rm M}_{\rm end}^{0-} = 190$ K, ${\rm R}_{\rm begin}^{0-} = 225$ K, ${\rm R}_{\rm end}^{0-} = 265$ K, $k_{\rm R} = 0.075$, $k_{\rm M} = 0.262$, which, according to [3], correspond to $\rm Ti_{50}Ni_{47}Fe_3$ alloy. Cooling is performed to the completely martensitic state, i.e., heating begins with the single-phase state. Nevertheless, in the case of relatively small stresses $\sigma = 100$ MPa $< \sigma_6$ (loop 1) the reverse

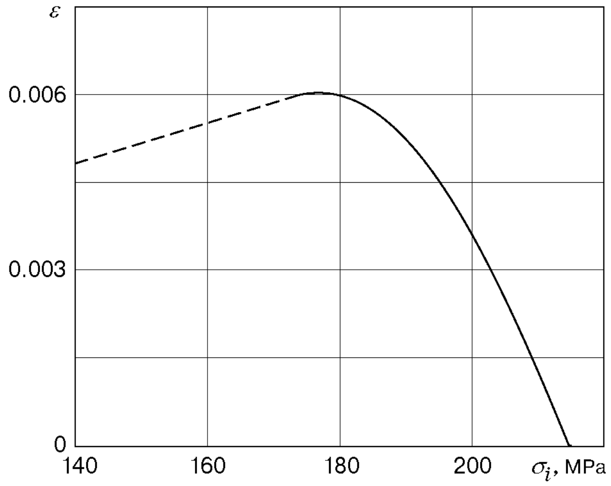

Fig. 5

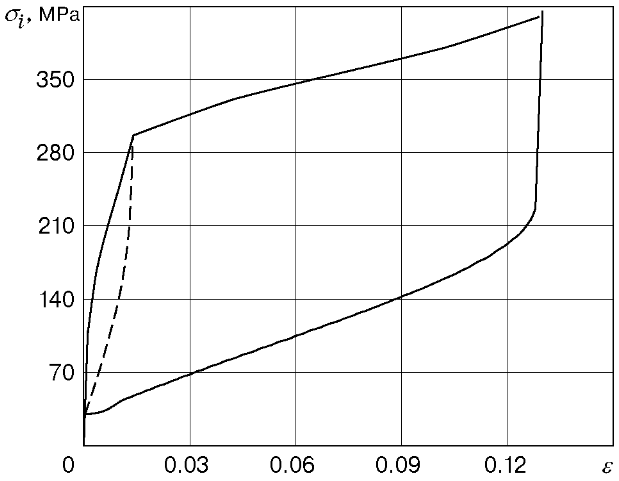

Fig. 6

transformation is a two-step process, and the steps separated by an anhysteretic segment, and the large-strain segment precedes the small-strain segment. At $\sigma = 200$ MPa (loop 2), the anhysteretic segments of the direct and reverse transitions decrease. Finally, at $\sigma = \sigma_6 = 350$ MPa (loop 3) the steps of the reverse transition do not differ.

Figure 5 shows a curve of the phase strain ε accumulated in the direct rhombohedral transformation under constant stress versus this stress at the transition temperatures (2.2). At $\sigma < \sigma_1 \approx 173$ MPa, the direct rhombohedral transformation occurs until complete transition from the austenitic phase to the rhombohedral phase, and ε increases linearly with increase in applied stress (dashed straight line in Fig. 5). With increase in stress in the region $\sigma_1 < \sigma < \sigma_2$, the temperature interval of the rhombohedral transformation and the degree of completeness of the R-transformation at the end of this region decrease (see Fig. 1) because this phase transition stops, without having completed, and the martensitic transformation begins. At the same time, because of increase in stress, the growth rate of ε increases. As a result, ε first increases with increase in stress and then drops rapidly to zero at $\sigma = \sigma_2 \approx 215$ MPa, because beginning with this value of stress, the rhombohedral transformation does not occur. Thus, the proposed model describes qualitatively the nonmonotonic change in the phase strain of the rhombohedral transformation under increase in stress [7].

The proposed model describes not only the direct and reverse transformations under the action of constant stress but also the phenomena of the martensitic and rhombohedral inelasticity and pseudoelasticity under isothermal active loading from the austenitic state. The corresponding hysteresis curves calculated from the formulas given above are shown in Fig. 6 for the characteristics of the material (2.2) and $T = 130$°C. The high hardening and the narrow loop correspond to the rhombohedral transformation caused by stresses, (the dashed curve corresponds to unloading), and the small hardening and the wide loop correspond to the transition martensitic, which agrees with experimental data of [2, 3].

This work was supported by the International Scientific and Technical Center (Grant No. 1536) and the Russian Foundation for Fundamental Research (Grant No. 99-01-01187).

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